Information cascades in the labor market*

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Abstract

A model of herding behavior in the labor market is presented where employers receive signals with limited precision about the workers' types, and can observe previous employers' decisions. Both the employer and the worker can influence the signal probabilities. In particular, the employer tries to increase the precision of the signal about the worker's type whereas the worker wants to get a good signal, independent of her type. In a two-period model, we derive conditions for an equilibrium in which only down-cascades occur, i.e., the second employer does not hire a worker with a bad history even if he receives a favorable private signal about the worker's type, but he follows his own signal if the worker's history is good.

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JEL codes: D83, J64

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1 Introduction

One of the leading economic examples for rational herd behavior is the labor market.[1] When a worker applies for jobs at different employers sequentially, current employers can infer something about the worker's abilities or "type" by observing previous employers' decisions. These decisions are summarized in the CV as spells of employment with particular employers, or spells of unemployment. While good jobs in the past imply that previous employers received favorable signals about the abilities of the worker, unemployment spells are attributed to the fact that applications failed, i.e., potential employers chose not to hire the worker. Thus, an applicant who receives good offers in the beginning of her career can become a "star" whereas a bad start without good job offers can make subsequent employers unwilling to hire a worker. In this sense, information cascades may dominate a worker's career.

When comparing the labor market interpretation of herding theories with most other applications of information cascade games, such as investment and lending decisions, one difference seems particularly striking. In contrast to investment projects or loans, workers who can become the object of cascades are able to react and adjust to this phenomenon. For example, a worker who knows that the beginning of her career is decisive for her future success will send out many applications early in her life, and put in a lot of effort to prepare for interviews, assessment centers etc., or to obtain helpful letters of recommendation. In this paper, we incorporate such efforts by allowing the worker to increase the probability that the employer gets a favorable impression of her. Also, we endogenize the employer's choice of the precision of the test for a new applicant. Both the applicant's and the employer's manipulations of the signal probabilities are assumed to create some cost to the agents. The extent to

[1] See, e.g., Bikhchandani, Hirshleifer, and Welch (1992) and the experimental study by Anderson and Holt (1997). For a survey see Bikhchandani, Hirshleifer, and Welch (1998).

which signal probabilities are influenced by the agents will depend on the employment history, and on the expectations about others' behavior. We investigate the effect of these choices on the occurrence of information cascades in the equilibrium of the game.

In the stylized two-period model specification we study below, the introduction of the signal manipulations by the worker and the employer causes an asymmetry between good and bad employment histories. In general, two types of cascades are possible in equilibrium, namely up-cascades where the second employer "blindly" employs the worker if she has been employed before and down-cascades where an employer does not hire a worker who was unemployed in the first period. But it is shown that only down-cascades can occur alone. I.e., for certain parameter ranges, early spells of unemployment are decisive for the worker's subsequent job search, but spells of employment are not. This means that the possibility of a worker to improve her chance of employment can make unemployment a strong negative stigma in equilibrium.

The main driving force of this asymmetry result is the following: Consider an equilibrium with a possible down-cascade. The worker knows that if she is not employed in the first period, she won't be employed in the second period. A good worker type then has a stronger incentive to generate a good signal than a bad worker type. The reason is that a bad worker type loses less from getting into a down-cascade, as she is less likely to get employed in the future anyway. Thus, bad signals come from bad worker types with a high probability, which reinforces the down-cascade. On the other hand, up-cascades are destabilized by the fact that bad worker types gain more from being in such a cascade than good worker types. They therefore exert high effort, which reduces the informativeness of a good signal.

In Section 2, the model and the main result are presented. The section also contains a numeric example which illustrates the size and ordering of the parameter ranges for which the different possible equilibria exist. Section 3 discusses the result in the context of related literature.

2 The model

2.1 Assumptions

Suppose there is one worker who can apply for a job in every period. In each period there is one employer with an open post. The game has two periods, $t = 1, 2$, but jobs last only one period and the worker cannot be reemployed by the same employer. When the firm hires a worker, it receives a return $V = 1$ if the worker is a good type (type G) and a return $V = 0$ if the worker is a bad type (type B). The wage payment to a worker, irrespective of her type, is 0.5. The prior probability of a good or a bad worker type is $\beta_1 = 0.5$. These parameters are chosen to make the employer indifferent between hiring and not hiring a worker in the absence of additional information.

The employer does not know the worker's type, but receives a signal about her ability, which can be either high ($S_t = H$) or low ($S_t = L$).[2] In each period t, signal $S_t = H$ and signal $S_t = L$ occur with the following probabilities, given the worker is a good ($V = 1$) or a bad ($V = 0$) type:

Table 1: Signal probabilities of the two worker types.

	V=1	V=0
$\mathrm{S}_t = H$	$0.5 + p_t + q_t^G$	$0.5 - p_t + q_t^B$
$\mathrm{S}_t = L$	$0.5 - p_t - q_t^G$	$0.5 + p_t - q_t^B$

The employer can influence the probability of receiving a good signal from a good type and a bad signal from a bad type (i.e., the precision of the signal) by choosing $p_t \in [0, \overline{p}]$, with some upper limit $\overline{p} < 0.5$. This costs him $K(p_t)$ with $K' > 0$, $K'' > 0$, $K(0) = 0$, $lim_{p_t \to \overline{p}} K(p_t) = \infty$, and $K'(0) = 0$. Without loss of generality we write

[2] The assumption of a binary signal is not without consequences. If the signal were continuous (more precise), the employer could set different cutoff values, depending on the employment history of the worker. However, herding could still occur in this case, as long as the employment decision is discrete (see the discussion in Section 3).

$K(p_t) = k \cdot \xi(p_t)$, in order to use k as a scaling parameter. The worker can influence the signal probabilities as well. She can choose q_t, with $q_t \in [0, \overline{q}]$, to increase the probability of a good signal, independently of her true type. However, different types may choose different effort levels when applying for a job, denoted by q_t^G and q_t^B. The cost function $C(q_t)$ is identical for both types, and increasingly convex, $C' > 0$, $C'' > 0$, $C''' \geq 0$, $C(0) = 0$, $lim_{q_t \to \overline{q}} C(q_t) = \infty$, and $C'(0) = 0$.[3] Again, we introduce a acaling parameter, c, by writing $C(q_t) = c \cdot \gamma(q_t)$. Furthermore, assume that $\overline{p} + \overline{q} \leq 0.5$, to guarantee interior solutions.

To allow for imperfectly correlated job profiles (e.g., workers switching to different kinds of jobs), we assume that a worker who was a good type in the first period may become a bad type in the second period, and vice versa. The probability of a good [bad] worker in period 1 of remaining a good [bad] worker in period 2 has a value of $\alpha \in [0.5, 1]$, which is common knowledge. E.g., if there is no correlation between the abilities required in period 1 and period 2, a good worker in period 1 is a good worker in period 2 with probability one half, i.e., $\alpha = 0.5$. Suppose further that before the second period starts neither the worker nor the employer know whether the worker's type changes, but in $t = 2$ the worker finds out whether she is a good or a bad type for the new job. The timing of the two-period game is as follows:

<u>Period $t = 1$:</u>

- The employer chooses p_t. Simultaneously, the worker learns her type, G or B, and chooses q_t.
- Firm 1 receives a signal S_t and either employs the worker or not. If employed, the worker receives the wage of 0.5 from the firm, and the firm gets the return V.

[3] The assumption that $C''' \geq 0$, which may be seen as rather restrictive, is only needed for one part of the main result, as will be specified below.

Period $t = 2$:

- The same as in $t = 1$, except that firm 2 learns the employment history of the worker, h_1, as well as p_1, before the period starts.

If the worker was employed in the first period, the history is denoted as $h_1 = 1$, and as $h_1 = 0$ if she was not employed.[4]

2.2 Optimal choices of the worker and the employer

To solve this game, we use the concept of Perfect Bayesian equilibrium. The worker determines her first-period effort by considering not only firm 1's behavior, but also what will happen in the second period after being employed or unemployed in the first period. In equilibrium, the worker knows whether the second employer will hire her after a good or a bad signal S_2, given her history. We say that a worker is in a cascade if in $t = 2$ she is employed (in an up-cascade) or not employed (in a down-cascade) *independent* of the signal that the new employer receives about her type. The following lemma characterizes the worker's effort choice.[5] (All proofs are relegated to the appendix.)

Lemma 1 *If the worker is in a cascade, she chooses* $q_t = 0$. *Otherwise, the optimal* $q_t^A, A = G, B$, *is given by*

$$C'(q_t^A) = 0.5 + U_{t+1}^A(H) - U_{t+1}^A(L) \tag{1}$$

with $U_{t+1}^A(S_t)$ *denoting the equilibrium continuation payoff of type* A *after signal* S_t *in period* t.[6]

[4] The assumption that the second firm learns the first firm's level of p_1 is made only for the sake of a clearer exposition. In all pure strategy equilibria of the game, p_1 is known anyway, and potential equilibria in which firm 1 uses mixed strategies can be excluded due to the fact that firm 1's profit is concave in p_2 (see the proof of Lemma 2 in the appendix).

[5] Both Lemma 1 and Lemma 2 hold for any finite number of periods t, although we will only consider two periods here.

[6] For this notation to apply to period $t = 2$, define $U_3^A(H) = U_3^A(L) = 0$.

From Lemma 1 it follows directly that when there is no cascade in period 2, the optimal effort in period 2 satisfies $C'(q_2^A) = 0.5$. This effort level will be denoted by q^*. In period 1, it holds that the higher the continuation payoff after a good signal and the lower the continuation payoff after a bad signal are, the more effort the worker exerts in the current period.[7]

Now consider the optimal choices of the firms. In the second period, the employer updates his beliefs about the worker's type based on whether she was employed in the first period or not. He chooses the precision of the signal, p_2, given his beliefs about the worker's type and his beliefs about the effort q_2 chosen by the worker.

Define β_t as the employer's prior probability of a good worker type in period t. Before period 1 we have $\beta_1 = 0.5$, which is then updated by the first employer after he receives the signal.

Lemma 2 *Firm t sets $p_t = p^*$ such that $K'(p^*) = 0.5$ and employs the worker after observing $S_t = H$ (and does not employ her after observing $S_t = L$) iff the following three conditions are satisfied:*[8]

$$\beta_t(0.5 - p^* - q_t^G) \leq (1 - \beta_t)(0.5 + p^* - q_t^B) \qquad \text{(non U)}$$

[7] For example, suppose the employer receives a good signal about the worker's type in period 1, he employs the worker, and no up-cascade starts. Then the continuation payoff for type G is given by

$$U_2^G(H) = \alpha[(0.5 + p_{2,(h_1=1)} + q^*)0.5 - C(q^*)] + (1 - \alpha)[(0.5 - p_{2,(h_1=1)} + q^*)0.5 - C(q^*)]$$

where $p_{2,(h_1=1)}$stands for the employer's choice in the second period after the worker was employed in the first period. The continuation payoff $U_2^G(L)$ is equal to $U_2^G(H)$ if there is no cascade after signal L (because without cascades the employer's choice of signal precision in the second period is independent of the worker's employment history, i.e., $p_{2,(h_1=1)} = p_{2,(h_1=0)}$, which will be shown below). Analogously, for type B, if no cascade starts after signal H,

$$U_2^B(H) = \alpha[(0.5 - p_{2,(h_1=1)} + q^*)0.5 - C(q^*)] + (1 - \alpha)[(0.5 + p_{2,(h_1=1)} + q^*)0.5 - C(q^*)]$$

which, again, is equal to $U_2^B(L)$ if there is no cascade after signal L.

[8] The worker types' respective choices used in the three conditions, q_t^G and q_t^B, are the workers' equilibrium choices, given the history in period t. It follows from Lemma 1 (and from the convexity of $C(\cdot)$) that each worker type's optimal choice is unique in a given equilibrium, so the employer knows q_t^G and q_t^B with certainty.

$$\beta_t(0.5 + p^* + q_t^G) \geq (1 - \beta_t)(0.5 - p^* + q_t^B) \quad \text{(non D)}$$

$$p^* + \beta_t q_t^G - (1 - \beta_t)q_t^B - 2K(p^*) \geq 0.5\,|\beta_t - (1 - \beta_t)| \quad \text{(C)}$$

Otherwise, the firm chooses $p_t = 0$, *employs the worker if* $\beta_t \geq 0.5$, *and does not employ the worker if* $\beta_t < 0.5$.

Conditions (*non U*) and (*non D*) are no-cascade conditions, ensuring that the employer prefers to follow his own signal, given p^*. The third condition, (C), requires the cost $K(p^*)$ to be sufficiently small to make investing into the signal precision worthwhile. Notice that the optimal p^* does not depend on β_t, q_t^G, q_t^B, nor on α. This simplifies the analysis considerably. Regarding the employer's choice in the first period, it holds that $\beta_1 = 0.5$, and hence the employer always chooses $p = p^*$ as all three conditions of Lemma 2 are satisfied. In particular, condition (C) reduces to $p^* - 2K(p^*) \geq 0.5(q_1^B - q_1^G)$, which is always satisfied as $K(p)$ is convex and the right-hand side is smaller or equal to zero for all equilibrium values of (q_1^G, q_1^B).[9] For the same reason, conditions (*non U*) and (*non D*) are also satisfied as they both reduce to $2p^* \geq q_1^B - q_1^G$.

2.3 Equilibria of the game

An equilibrium of this game specifies the agents' behavior after any employment history. In particular, it is possible that the second employer is prescribed to herd behind the first employer's decision only after one of the two possible employment histories, but not after the other. E.g., we use the term "equilibrium with up-cascades only" if firm 2 hires the worker after a history $h_1 = 1$, regardless of his own signal, but follows his own signal signal if $h_1 = 0$. This behavior might arise if the second employer considers a good employment history to be more informative than a bad one. By

[9] In all possible equilibria, $q_1^G \geq q_1^B$, which will be shown below.

analogy, there are three more possible pure-strategy equilibria, characterized by firm 2's behavior: Equilibria with down-cascades only (firm 2 follows firm 1 only after the worker was unemployed), equilibria with up-cascades and down-cascades (firm 2 follows firm 1's employment decision unconditionally), and equilibria with no cascades (firm 2 always follows its own signal). Of course, there is also the possibility of mixed-strategy equilibria, where, e.g., firm 2 follows firm 1 only with some probability. Also, a multiplicity of equilibria can arise.

The following proposition shows that there is a general asymmetry between up-cascades and down-cascades in the set of equilibria of the game. (The proposition takes as given the cost functions $C(q_t)$ and $\xi(p_t)$, as well as $\overline{q}$ and $\overline{p}$, and views α and k as parameters.)

Proposition 1 *(a) There does not exist an equilibrium with up-cascades only, for any parameter constellation. More generally, in any (mixed-strategy) equilibrium in which the history* $h_1 = 1$ *leads to an up-cascade with positive probability, the reverse history* $h_1 = 0$ *must lead to a down-cascade.*

(b) For k *sufficiently small and* α *sufficiently close to* 1*, there exists an equilibrium with down-cascades only. It is the unique equilibrium in pure strategies.*[10]

The workers' ability to influence the signal probabilities causes this asymmetry between up-cascades and down-cascades, as good signals become less informative than bad signals. In other words, if both workers strive to leave a good impression, the employer's likelihood of facing a good worker after receiving a good signal is lower than the likelihood of facing a bad worker after receiving a bad signal. This effect is reinforced when cascades are possible in equilibrium, because then the signal in the first period becomes more important and the workers increase their efforts further.

[10] For part (b) of the proposition, the assumption $C''' \geq 0$ is used.

A second driving force for the result is the fact that good and bad worker types choose different effort levels when they expect cascades to occur (see Lemma 1). In particular, a good worker loses more from being in a down-cascade than a bad worker, because without a cascade the good worker would be more likely to be employed in the second period. Therefore, a good type chooses a higher effort than a bad type in the first period, $q_1^G > q_1^B$, in the equilibrium with down-cascades only. This implies that a bad signal must come from a bad type with an even higher probability, leading to the second employer's rejection of an applicant who was previously unemployed. Conversely, equilibria with up-cascades only are destabilized by the same logic. In any such proposed equilibrium with up-cascades only, the bad worker would have more to gain from a good signal in $t = 1$ than the good worker. Hence, $q_1^B > q_1^G$ would hold, making a good signal even less informative, and the equilibrium breaks down.

If the correlation between jobs is weak, i.e., α is close to 0.5, information cascades can disappear completely. A reduction of the worker's cost of influencing the signal, c, strengthens the observed asymmetry between up-cascades and down-cascades. And if the employer's cost of increasing the signal precision is high (large k), equilibria with up-cascades and down-cascades are likely to exist. This is demonstrated in the example below.

2.4 An example

In this subsection, an example is presented for which we characterize the equilibria over the whole parameter range.[11] For this example we suppose $C(q_t) = c[\frac{1}{\overline{q}-q_t} - (\frac{1}{\overline{q}})^2 q_t - \frac{1}{\overline{q}}]$ and similarly $K(p_t) = k[\frac{1}{\overline{p}-p_t} - (\frac{1}{\overline{p}})^2 p_t - \frac{1}{\overline{p}}]$, which are cost functions satisfying the above assumptions. Also, we set $\overline{q} = \overline{p} = 0.25$ and $c = 0.01$. Then, the equilibrium ranges depending on α and k are as illustrated in Figure 1.

[11] The calculation is sketched in the appendix.

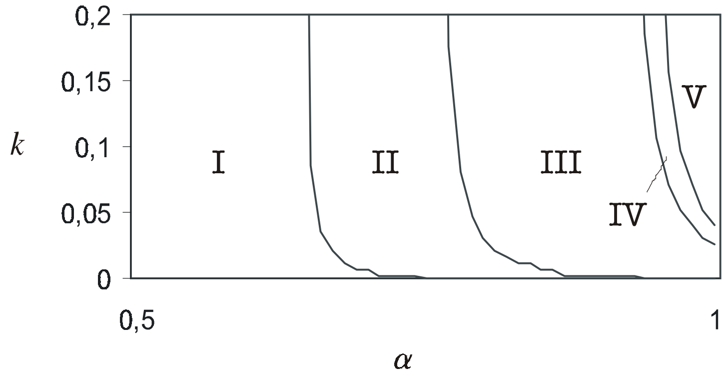

Figure 1: Parameter ranges of equilibria, with $c = 0.01$. Note: Region I: Unique equilibrium with no cascades. II: Several equilibria exist; one with no cascades, one with down-cascades only, and mixed equilibria. III: Unique equilibrium with down-cascades only. IV: Only mixed equilibria with a down-cascade after $h_1 = 0$, and a possible up-cascade after $h_1 = 1$. V: Unique equilibrium with up-and-down-cascades.

Consider Figure 1 and first focus on the case where $\alpha = 1$, i.e., the same abilities are required in the first and in the second period. In this case, there are two possible equilibria in pure strategies. If the employer's cost parameter k is below a critical level, the equilibrium is characterized by down-cascades only (region III), whereas if the employer's costs are high, both up-cascades and down-cascades occur (region V). Between these two areas, there are only mixed equilibria in which the second-period employer randomizes between $p_2 = p^*$ and $p_2 = 0$ with probability s and $(1 - s)$, respectively, if the worker was employed in the first period.[12]

Moving from $\alpha = 1$ to the left first lets the equilibrium with up-cascades and down-cascades disappear: If the job requirements are less strongly correlated, the second employer does not want to follow all of his predecessor's decisions even if the interview costs are high. The same argument also eliminates the equilibrium with down-cascades for low α, such that only an equilibrium with no cascades exists when job requirements

[12] The worker types choose effort levels contingent on s in these equilibria. For a clearer exposition, the formulation of Lemma 1 in Subsection 2.2 only applies to pure-strategy equilibria. Optimal behavior in mixed-strategy equilibria, which is analogous, is addressed in the appendix.

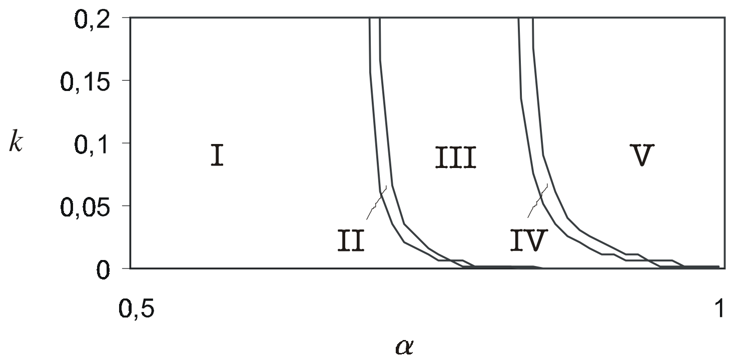

Figure 2: Parameter ranges of equilibria, with $c = 0.05$. Note: See Figure 1.

are barely correlated over time. Between the two ranges of unique equilibria, there are parameter values (region II) for which both an equilibrium with down-cascades only and an equilibrium with no cascades exists, as well as mixed equilibria in which a bad history induces a down-cascade with some positive probability.

An increase in the applicant's cost of influencing the signal probability makes the asymmetric equilibria disappear for some parameter combinations. Figure 2 shows the equilibrium ranges for $c = 0.05$. There, regions II and III, in which the equilibrium with only down-cascades exists, are both smaller than in Figure 1. Also, it is evident that equilibria with up-cascades and down-cascades occur in a much larger parameter range (region V). Signals are less distorted due to higher costs, so the second employer learns more from the applicant's employment history.

3 Discussion

The paper analyzes the effect of endogenous signal probabilities on information cascades in a labor market setting. In contrast to the benchmark model developed by Bikhchandani, Hirshleifer, and Welch (1992), which corresponds to the limit case $\{\alpha = 1, \overline{q} = 0,$ and $k = 0\}$, cascades can start already in the second period, and down-cascades (i.e., cascades in which a worker is not employed) are more likely to exist than up-cascades (i.e., cascades in which a worker is employed).

It is important to notice that the model presented here makes very specific assumptions (e.g., on the worker types' prior probabilities, on the signal space, etc.), in order to derive the result in a simple way. Many variations of the model seem possible, and it is natural to check the robustness of the results. We will shortly discuss two possible extensions, namely the case of a continuous signal space and worker competition.

In models with continuous signal spaces, the employers will optimally choose cutoff values for the signals (depending on the history), and an applicant will be employed if and only if her signal lies above the cutoff. If applicants can influence the signal distributions, employers will adjust these cutoff values optimally. Hence, contrary to the binary-signal case discussed here, it is no longer true that "bad" signals occur less often and are more informative than good signals. However, cascades are still possible of course, since the employers may prefer to choose a very high or very low cutoff, and hence decide irrespective of the signal realization. Also, the asymmetry derived in our model will still be relevant, as the second driving force of the result is still valid: If facing the threat of a down-cascade, good worker types will put in more effort than bad worker types, because they have more to lose from a down-cascade. The worker selection in early periods therefore becomes more informative, and down-cascades are self-enforcing even in models with continuous signals. Conversely, if only up-cascades were possible, bad types would exert higher additional effort than good types, since they would have more to gain from up-cascades. Hence, the selection of workers in early periods would be relatively uninformative, and equilibria with up-cascades would be destabilized.

Now consider the case of two workers competing for a single job. If a given applicant was rejected in the first period of such a game, the competition will make this rejection less informative, since it is possible that both workers produced good signals, but only one could be employed. Hence, the second employer learns less about a rejected applicant's type, and down-cascades are less likely when workers compete. However, the incentive effect described above still applies, as good workers lose more in a down-cascade than bad workers. Consider a game starting with a high prior probability of a bad worker type. In this case, it is easily conceived that the worker types' optimal adjustment to the possibility of cascades can make up-cascades disappear or induce down-cascades.

Literature

There are a number of articles offering an information-based rationale for the finding that the re-employment probability and the wage offered depend negatively on the duration of unemployment.[13] First, there is a small literature on social learning, wages, and hiring decisions. Within the framework of matching models, Stern (1990) and Lockwood (1991) analyze how firms use the information conveyed by other firms' hiring decisions and condition their decision on the applicant's employment history. Stern shows that wage offers fall with the length of an applicant's unemployment spell. Lockwood derives acceptable periods of unemployment, as the firms in his model choose a cutoff time after which they ignore their private information and never hire a worker. Also, Gibbons and Murphy (1992) and Farber and Gibbons (1996) model how Bayesian learning influences wages as unobservable abilities of workers are revealed over time. Gibbons and Katz (1991) provide a related empirical analysis of employment and layoff data using a model of asymmetric information about worker ability. Our model differs from all of these models in that we restrict wages to be fixed, but allow workers to optimize, not only firms.

[13] Other explanations are that the search intensity of unemployed persons declines over time or that human capital becomes obsolete.

A second set of results relevant to this paper comes from a field experiment conducted by Oberholzer-Gee (2000), who focuses on the effect of unemployment spells on applicants' re-employment probabilities. His results indicate that in Switzerland, a person who was unemployed for two and a half years is 47% less likely to be hired than an employed person, while in the U.S. no unemployment stigma can be observed. Within our model, this would be consistent with people performing rather similar jobs during their lifetime in Switzerland, as compared to the United States where people are relatively more likely to switch to jobs requiring a completely different set of abilities. Therefore, social learning may be much more important in the Swiss labor market than in the U.S., and spells of unemployment may be more informative in Switzerland than in the U.S.[14]

From another point of view, the model can be interpreted as a rational theory consistent with the so-called "negativity effect", which is well-documented in the psychological literature.[15] This effect refers to the greater weight placed on negative information relative to positive information. In our setup with endogenous signal qualities it can be an equilibrium that employers do not hire a worker who was unemployed regardless of their own signal, but follow their own signal if the worker was employed previously. In this sense, previous unemployment may be weighed more than previous employment when assessing an applicant's value.

Finally, consider some policy implications of the derived asymmetry between up- and down-cascades. The fact that down-cascades play a prominent role when signal probabilities are endogenous might justify labor market policies to stop such cascades entailing bad jobs and/or unemployment. For example, a worker trapped in a down-

[14] An alternative explanation of the results, put forward by Oberholzer-Gee, is that wage subsidies and other measures for the unemployed in Switzerland create a strong stigma for those who still cannot find a job.

[15] See especially Fiske (1980) who uses hiring practices as her leading example and presents evidence that mainly negative cues influence the evaluation of people.

cascade can be retrained. A new profession allows a good worker to start a new career as the previous employment history becomes less informative. Also, new favorable public information about a worker, e.g. a diploma or a degree, can be strong enough to outweigh the negative information that has led into the cascade. More generally, the observed asymmetry can be responsible for an income distribution with a thick lower tail. Thus, the distribution effects caused by the asymmetry identified above might be unwelcome from a social welfare perspective.

Appendix

Proof of Lemma 1: First, when the worker is in a cascade, she will not exert any effort as the employer will not follow his own signal. Second, the expected utility of a good worker type is

$$U_t^G = (0.5 + p_t + q_t^G)(0.5 + U_{t+1}^G(H)) + (0.5 - p_t - q_t^G)U_{t+1}^G(L) - C(q_t^G)$$

(and similarly of a bad type after switching the sign in front of p_t). Taking the first derivative with respect to q_t yields equation (1). ■

Proof of Lemma 2: The optimal p_t is found by maximizing the employer's expected profit,

$$\Pi(p_t) = \beta_t(0.5 + p_t + q_t^G)0.5 + (1 - \beta_t)(0.5 - p_t + q_t^B)(-0.5) - K(p_t),$$

with respect to p_t, yielding p^*. The first two conditions, (*non U*) and (*non D*), ensure that the employer neither follows his prior belief if the worker was employed, nor if the worker was unemployed, but follows his own signal, given that p^* was chosen. The third condition, (C), is derived by comparing the employer's profit from setting $p_t = 0$ or $p_t = p^*$, respectively. If $\beta_t \geq 0.5$, $\Pi(p^*) \geq \Pi(0)$ iff

$$0.5(p^* + \beta_t q_t^G - (1 - \beta_t)q_t^B) - K(p^*) \geq 0.25(\beta_t - (1 - \beta_t)).$$

If $\beta_t < 0.5$, $\Pi(p^*) \geq \Pi(0)$ iff

$$0.5(\beta_t - (1-\beta_t)) + p^* + \beta_t q_t^G - (1-\beta_t) q_t^B - 2K(p^*) \geq 0.$$

Combining these two inequalities yields condition (C).

Proof of the Proposition: (a) In the equilibrium with up-cascades only, the worker chooses q_1^A such that $C'(q_1^A) = 1 - U_2^A(L)$. Since a good worker's continuation payoff after a bad signal in period 1 is higher than that of a bad worker, $U_2^G(L) > U_2^B(L)$, it follows that $q_1^B > q_1^G$ (and both exceed q^*). In the second period, the worker exerts no effort after being employed in the first period, $q_{2,(h_1=1)}^G = q_{2,(h_1=1)}^B = 0$, but chooses some effort after not being employed, $q_{2,(h_1=0)}^G = q_{2,(h_1=0)}^B = q^*$. The employer sets $p_1 = p_{2,(h_1=0)} = p^*$ and $p_{2,(h_1=1)} = 0$.

First consider a worker with a good history, $h_1 = 1$. The employer's updated prior is $\beta_2 = \frac{\alpha(0.5+p^*+q_1^G)+(1-\alpha)(0.5-p^*+q_1^B)}{1+q_1^G+q_1^B}$. Suppose that firm 2 considers a deviation, i.e., to set $p_{2,(h_1=1)} = p^*$ and to follows its own signal (as the best possible deviation). For the equilibrium to exist, this deviation must not be profitable, which implies that either (C) or $(non\ U)$ or $(non\ D)$ must be violated. Condition $(non\ U)$ requires that $\beta_2(0.5 - p^* - q^*) \leq (1-\beta_2)(0.5 + p^* - q^*)$. Note that β_2 is increasing in α. (This follows from $0.5 + p^* + q_1^G \geq 0.5 - p^* + q_1^B$, which must hold in equilibrium, because otherwise a good signal would indicate a bad worker, implying $q_1^G = q_1^B = 0$ as a best response.) Thus, if $(non\ U)$ holds for $\alpha = 1$, it must always hold. Some manipulations together with $\alpha = 1$ yield $-q^*(2p^* + q_1^G + q_1^B) \leq (0.5 + p^*)q_1^B - (0.5 - p^*)q_1^G$, which is always true (because $q_1^B > q_1^G$). As $(non\ D)$ holds also after $h_1 = 1$, it follows that the deviation is profitable iff (C) is true.[16] Condition (C) demands that

$$\frac{p^* - 2K(p^*)}{2\alpha - 1} \geq (0.5 - q^*)\frac{2p^* + q_1^G - q_1^B}{1 + q_1^G + q_1^B}. \tag{2}$$

[16] In all equilibria, condition $(non\ D)$ $[(non\ U)]$ is trivially satisfied after history $h_1 = 1$ $[h_1 = 0]$.

Towards a contradiction, suppose that this inequality is violated, which means that an up-cascade exists. But now consider whether this is consistent with no down-cascades after $h_1 = 0$. In particular, after $h_1 = 0$, condition (C) must be satisfied for down-cascades not to exist:

$$\frac{p^* - 2K(p^*)}{2\alpha - 1} \geq (0.5 + q^*)\frac{2p^* + q_1^G - q_1^B}{1 - q_1^G - q_1^B}. \tag{3}$$

If (2) is violated, then (3) must be violated, too. This implies that an equilibrium with up-cascades only does not exist.

Now consider equilibria in mixed strategies. Suppose that the second employer randomizes after history $h_1 = 1$, resulting in a positive probability of an up-cascade. From Lemma 2, the only possibility for randomization in $t = 2$ to be optimal is that condition (C) holds and the employer randomizes between setting $p = 0$ and $p = p^*$. (He would follow firm 1's decision in the former case, and his own signal in the latter case.) The worker chooses her effort levels on both stages conditional on the employer's probabilities to set $p = p^*$. With any combination of the the worker types' effort levels, $(q_1^G, q_1^B, q_{2,(h_1=1)}^G, q_{2,(h_1=1)}^B, q_{2,(h_1=0)}^G, q_{2,(h_1=0)}^B)$, it is straightforward to check that if condition (C) holds with equality after $h_1 = 1$, it must be violated after $h_1 = 0$ (in analogy to the comparison of (2) and (3), above). Hence, it is impossible that the second employer follows his own signal after $h_1 = 0$, with any positive probability.

(b) We first show that the equilibrium exists in the limit, as $\alpha \to 1$ and $k \to 0$. Existence of an equilibrium with down-cascades only requires that all three conditions of Lemma 2 are satisfied for $h_1 = 1$ and that at least one of them is violated for $h_1 = 0$. The worker chooses $q_1^G > q_1^B$ $(> q^*)$ in the first period. In the second period she sets $q_{2,(h_1=1)}^G = q_{2,(h_1=1)}^B = q^*$ after being employed in the first period and $q_{2,(h_1=0)}^G = q_{2,(h_1=0)}^B = 0$ if she was not employed. The employer tests the worker in the first period and in the second period after a good history, but not after she was unemployed: $p_1 = p_{2,(h_1=1)} = p^*$ and $p_{2,(h_1=0)} = 0$. After a good history $h_1 = 1$, the second employer's updated prior for a good type is $\beta_2 = ((2\alpha-1)p^* + 0.5 + \alpha q_1^G + (1-\alpha)q_1^B)/(1+q_1^G+q_1^B)$. Condition (*non U*) then requires $\beta_2(0.5 - p^* - q^*) \leq (1-\beta_2)(0.5+p^*-q^*)$. Using the fact that $\beta_2 \geq 0.5$, it is sufficient for this condition to hold that $\beta_2 \leq p^* + 0.5$, which can be reformulated, with $\alpha \to 1$ and $k \to 0$ (the latter implying that $p^* \to \overline{p}$), as

$$\frac{q_1^G}{q_1^G + q_1^B} \leq \overline{p} + 0.5. \tag{4}$$

Using Lemma 1, we have $C'(q_1^B) > 0.5$. Together with $p^* < \overline{p}$ and the fact that $C'(q_1^G) - C'(q_1^B) = p^*$ holds in this equilibrium (again from Lemma 1), this implies $C'(q_1^B) > \frac{1}{2\overline{p}}(C'(q_1^G) - C'(q_1^B))$. Since C' is an increasing convex function ($C'' > 0$ and $C''' \geq 0$), it follows that

$$q_1^B > \frac{1}{2\overline{p}}(q_1^G - q_1^B). \tag{5}$$

Reformulating this to $q_1^B > q_1^G/(2\overline{p}+1)$ and replacing q_1^B in (4) yields as a sufficient condition for (*non U*) that $2\overline{p} + 2 \geq 2$, which is always satisfied. Next, examine condition (C) after $h_1 = 1$, which must also hold in an equilibrium with down-cascades only,

$$p^* - 2K(p^*) \geq (2\alpha - 1)(0.5 - q^*)\frac{2p^* + q_1^G - q_1^B}{1 + q_1^G + q_1^B}.$$

For $\alpha \to 1$ and $k \to 0$ this will hold if $\overline{p} \geq 0.5(2\overline{p} + q_1^G - q_1^B)/(1 + q_1^G + q_1^B)$, which is equivalent to $q_1^G + q_1^B \geq (q_1^G - q_1^B)/(2\overline{p})$. The latter is always satisfied, as it is implied by inequality (5).

After a bad history, $h_1 = 0$, either (*non D*) or (C) must be violated. Consider (C):

$$p^* - 2K(p^*) \geq (2\alpha - 1)(0.5 + q^*)\frac{2p^* + q_1^G - q_1^B}{1 - q_1^G - q_1^B}$$

For $\alpha \to 1$ and $k \to 0$, this becomes $\overline{p} \geq (0.5 + q^*)(2\overline{p} + q_1^G - q_1^B)/(1 - q_1^G - q_1^B)$. A simple rearrangement leads to $2\overline{p}(0.5 - 0.5q_1^G - 0.5q_1^B) \geq (2\overline{p} + q_1^G - q_1^B)(0.5 + q^*)$, which can never be satisfied. Thus, the equilibrium with down-cascades only exists.

To show uniqueness, we check the two other possible pure-strategy equilibria (for large α and small k), starting with the equilibrium with no cascade. In such an equilibrium, there is no learning. Thus, the worker and the employers choose the same optimal effort level in each period, $q_t = q^*, p_t = p^*, t = 1, 2$. Suppose the worker was not employed in the first period, $h_1 = 0$. Then, the second employer's updated probability for a good type is $\beta_2 = \frac{0.5 - q^* + p^*(1-2\alpha)}{1-2q^*}$, after some reformulations. Condition (C) of Lemma 2 then requires $p^* - 2K(p^*) \geq (2\alpha - 1)(0.5 + q^*)(2p^*)(1 - 2q^*)$. For $\alpha \to 1$ and $k \to 0$, this condition can hold only if $\overline{p}(1 - 2q^*) > \overline{p}(1 + 2q^*)$,which is never satisfied. Thus, an equilibrium with no cascades does not exist.

Now consider whether there is an equilibrium with up-cascades and down-cascades. Both worker types set q_1^A such that $C'(q_1^A) = 1$, denoted by q^{**}, and $q_2^A = 0$. The employer chooses $p_1 = p^*$ and $p_2 = 0$. This equilibrium does not exist if the employer sets $p_2 = p^*$ after some history h_1, i.e., if all three conditions of Lemma 2 are satisfied for either $h_1 = 1$ or $h_1 = 0$. For $h_1 = 1$, (*non* U) must hold, i.e., $(0.5 + q^{**} + (2\alpha - 1)p^*)(0.5 - p^*) \leq (0.5 + q^{**} - (2\alpha - 1)p^*)(0.5 + p^*)$, which is satisfied for all α. Also, (*non* D) holds, such that (C) is the only condition remaining to be checked. With $\beta_2 = (0.5 + q^{**} + p^*(2\alpha - 1))/(1 + 2q^{**})$, it is given by $p^* - 2K(p^*) \geq (2\alpha - 1)p^*/(1 + 2q^{**})$, which for $\alpha \to 1$ and $k \to 0$ becomes $\overline{p} \geq \overline{p}/(1 + 2q^{**})$. This is always satisfied, implying that the equilibrium with up-cascades and down-cascades does not exist.■

Calculation of the example: In order to generate Figures 1 and 2, Lemmas 1 and 2 are used in a similar way as in the proof of the proposition. In particular, the four lines in the figure are given by, from left to right, condition (C) after $h_1 = 0$ for the equilibrium with down-cascades, condition (C) after $h_1 = 0$ for the equilibrium without cascades, condition (C) after $h_1 = 1$ for the equilibrium with down-cascades, and condition (C) after $h_1 = 1$ for the equilibrium with up- and down cascades. It can be shown that these are the binding conditions for the pure-strategy equilibria to exist, dividing the parameter space into different equilibrium ranges.

No equilibrium in pure strategies exists if

$$\frac{2p^*}{1+2q^{**}} \leq \frac{p^* - 2K(p^*)}{(0.5 - q^*)(2\alpha - 1)} < \frac{2p^* + q_1^G - q_1^B}{1 + q_1^G + q_1^B} \tag{6}$$

(where q^* and q^{**} satisfy, as above, $C'(q^*) = 0.5$ and $C'(q^{**}) = 1$, respectively, and q_1^G and q_1^B are the worker types' effort levels in the equilibrium with down-cascades only, in $t = 1$). In an according equilibrium in mixed strategies (region IV), the firm has to be indifferent between setting $p = 0$ and $p = p^*$ if $h_1 = 1$. This is the case if condition (C) holds with equality,

$$\frac{p^* - 2K(p^*)}{(0.5 - \widetilde{q}_{2,(1)}(s))(2\alpha - 1)} = \frac{2p^* + \widetilde{q}_1^G(s) - \widetilde{q}_1^B(s)}{1 + \widetilde{q}_1^G(s) + \widetilde{q}_1^B(s)} \tag{7}$$

where s is the probability of the employer choosing $p = p^*$, $\widetilde{q}_1^G(s)$ and $\widetilde{q}_1^B(s)$ are the the worker types' optimal effort levels, given s, in $t = 1$, and $\widetilde{q}_{2,(1)}(s)$ is the effort of both worker types in $t = 2$, after $h_1 = 1$. Analogous to Lemma 1, these optimal effort levels are determined by

$$\begin{aligned}
C'(\widetilde{q}_1^G(s)) &= 0.5 + s(0.5 + p^* + \widetilde{q}_2(s))0.5 + (1-s)0.5 - C(\widetilde{q}_2(s)), \\
C'(\widetilde{q}_1^B(s)) &= 0.5 + s(0.5 - p^* + \widetilde{q}_2(s))0.5 + (1-s)0.5 - C(\widetilde{q}_2(s)), \\
\text{and } C'(\widetilde{q}_{2,(1)}) &= s \cdot 0.5.
\end{aligned}$$

One can then check numerically that there is always a probability s such that the optimal $\widetilde{q}_{2,(1)}(s)$, $\widetilde{q}_1^G(s)$, and $\widetilde{q}_1^B(s)$ fulfill equality (7), given the assumed cost functions. This ensures existence of a mixed equilibrium in this region. The mixed equilibria in region II are derived analogously.

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